Int J Theor Phys (2009) 48: 3498–3509
DOI 10.1007/s10773-009-0154-3

# Anti-synchronization of Time-delayed Chaotic Neural Networks Based on Adaptive Control

**Choon Ki Ahn**

Received: 4 July 2009 / Accepted: 28 September 2009 / Published online: 8 October 2009


**Abstract** This paper investigates the adaptive anti-synchronization problem for time-delayed chaotic neural networks with unknown parameters. Based on Lyapunov-Krasovskii stability theory and linear matrix inequality (LMI) approach, the adaptive anti-synchronization controller is designed and an analytic expression of the controller with its adaptive laws of unknown parameters is shown. The proposed controller can be obtained by solving the LMI problem. An illustrative example is given to demonstrate the effectiveness of the proposed method.

**Keywords** Anti-synchronization · Delayed chaotic neural networks · Adaptive control · Linear matrix inequality (LMI) · Lyapunov-Krasovskii stability theory

## 1 Introduction

Synchronization is a fundamental phenomenon that enables coherent behavior in coupled dynamical systems. Since the discovery of chaos synchronization by Pecora and Carroll [1], there have been tremendous interests in studying the synchronization of various chaotic systems. It has been widely explored in a variety of fields including physical, chemical and ecological systems [2]. Another interesting phenomenon discovered was the anti-synchronization, which is noticeable in periodic oscillators. The anti-synchronization, which is the vanishing of the sum of the relevant state variables of synchronized systems, has been investigated both experimentally and theoretically in many physical systems [3–7]. A recent study of the anti-synchronization phenomenon in non-equilibrium systems suggests that the anti-synchronization could be exploited as a technique for particle separation in a mixture of interacting particles [7]. There have been trials on applying some control methods to anti-synchronize chaotic systems. In [5], a linear controller was constructed for

This paper was supported by Wonkwang University in 2009.

C.K. Ahn (✉)
Division of Electronic and Control Engineering, Wonkwang University, Iksan, Korea
e-mail: hironaka@wonkwang.ac.kr

anti-synchronizing coupled identical chaotic systems. Nonlinear anti-synchronization controllers for nonlinear gyros and non-identical chaotic ratchets were proposed in [6] and [7], respectively. Recently, Al-sawalha and Noorania [8] constructed nonlinear controllers for anti-synchronization between two identical and different chaotic systems.

Time-delay often appears in many physical systems such as aircraft, chemical, and biological systems. Unlike ordinary differential equations, time-delayed systems are infinite dimensional in nature and time-delay is, in many cases, a source of instability. The stability issue and the performance of time-delayed systems are, therefore, both of theoretical and practical importance. Since Mackey and Glass [9] first found chaos in time-delay system, there has been increasing interest in time-delay chaotic systems [10, 11]. In this regard, some control methods, such as guaranteed cost control [12, 13], delayed feedback control [14], neural network approach [15], and impulsive control [16, 17], were proposed for synchronizing time-delayed chaotic systems.

In practical engineering situations, parameters are probably unknown and may change from time to time. When chaotic systems have some unknown parameters, it is generally difficult to synchronize the chaotic systems. In this case, it is well known that the adaptive control scheme is an effective method for the chaos synchronization. Thus, knowledge of the adaptive synchronization for chaotic systems with unknown parameters is of considerable practical importance. In this regard, the adaptive synchronization for unknown chaotic systems has been investigated by several researchers [18–22]. Recently, controllers for the adaptive anti-synchronization were proposed in [23, 24]. These works were restricted to chaotic systems without time-delay. To the best of our knowledge, however, for the adaptive anti-synchronization of time-delayed chaotic systems, there is no result in the literature so far, which still remains open and challenging.

Motivated by the above discussion, our main aim in this paper is to shorten this gap by investigating the adaptive anti-synchronization problem of time-delayed chaotic neural networks with unknown parameters. A new controller with its adaptive laws of unknown parameters for the adaptive anti-synchronization of time-delayed chaotic neural networks is proposed based on Lyapunov-Krasovskii stability theory [25, 26] and linear matrix inequality (LMI) approach. This controller is a new contribution to the topic of anti-synchronization. The proposed controller can be obtained by solving the LMI problem. The LMI problem can be solved efficiently by using recently developed convex optimization algorithms [27].

This paper is organized as follows. In Sect. 2, we formulate the problem. In Sect. 3, an LMI problem for the adaptive anti-synchronization of delayed chaotic neural networks is proposed. In Sect. 4, a numerical example is given, and finally, conclusions are presented in Sect. 5.

## 2 Problem Formulation

Consider a class of uncertain time-delayed chaotic neural networks

$$\begin{aligned}\dot{x}(t) = Ax(t) + \bar{A}x(t-\tau) + Bf(x(t)) + \bar{B}g(x(t-\tau))\\ + \sum_{k=1}^{p} \Phi_k(x(t))\theta_k + \sum_{l=1}^{q} \Psi_l(x(t-\tau))\phi_l,\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $\tau > 0$ is the time-delay, $A \in R^{n\times n}$ is the self-feedback matrix, $\bar{A} \in R^{n\times n}$ is the delayed self-feedback matrix, $B \in R^{n\times n}$ is the connection weight matrix, $\bar{B} \in R^{n\times n}$ is the delayed connection weight matrix, $f(x(t)) : R^n \to R^n$ and $g(x(t)) : R^n \to R^n$ are activation function vectors satisfying the global Lipschitz conditions with Lipschitz constants $L_f > 0$ and $L_g > 0$, respectively, $\Phi_k(x(t))$ $(k = 1, \dots, p) : R^n \to R^{n\times r}$ and $\Psi_l(x(t))$ $(l = 1, \dots, q) : R^n \to R^{n\times s}$ are nonlinear function matrices, and $\theta_k \in R^r$ $(k = 1, \dots, p)$ and $\phi_l \in R^s$ $(l = 1, \dots, q)$ represent the unknown constant parameter vectors. The system (1) is considered as a drive system. The synchronization problem of system (1) is considered by using the drive-response configuration. According to the drive-response concept, the controlled response system is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \bar{A}\hat{x}(t-\tau) + Bf(\hat{x}(t)) + \bar{B}g(\hat{x}(t-\tau)) + u(t), \tag{2}$$

where $\hat{x}(t) \in R^n$ is the state vector of the response system and $u(t) \in R^n$ is the control input. Define the anti-synchronization error $e(t) = \hat{x}(t) + x(t)$. Then we obtain the anti-synchronization error system

$$\begin{aligned}\dot{e}(t) &= Ae(t) + \bar{A}e(t-\tau) + B(f(\hat{x}(t)) + f(x(t))) + \bar{B}(g(\hat{x}(t-\tau)) + g(x(t-\tau))) \\ &\quad + \sum_{k=1}^{p} \Phi_k(x(t))\theta_k + \sum_{l=1}^{q} \Psi_l(x(t-\tau))\phi_l + u(t).\end{aligned} \tag{3}$$

Throughout this paper, we define that $\hat{\theta}_k(t)$ $(k = 1, \dots, p)$ and $\hat{\phi}_l(t)$ $(l = 1, \dots, q)$ are the estimate values of $\theta_k$ and $\phi_l$, respectively.

**Definition 1** (Adaptive Anti-synchronization) The error system (3) is adaptively anti-synchronized if the anti-synchronization error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0 \tag{4}$$

under the control input $u(t)$ with the adaptive laws $\hat{\theta}_k(t)$ and $\hat{\phi}_l(t)$ $(k = 1, \dots, p,\ l = 1, \dots, q)$.

The purpose of this paper is to design the feedback control input $u(t)$ guaranteeing the adaptive anti-synchronization. In order to design the feedback control input $u(t)$, we need information on $x(t)$, $\hat{x}(t)$, $\hat{\theta}_k(t)$, and $\hat{\phi}_l(t)$ $(k = 1, \dots, p,\ l = 1, \dots, q)$. Thus, the control input $u(t)$ in (2) depends on $x(t)$, $\hat{x}(t)$, $\hat{\theta}_k(t)$, and $\hat{\phi}_l(t)$ $(k = 1, \dots, p,\ l = 1, \dots, q)$.

## 3 Main Result

In this section, we present the LMI problem for achieving the adaptive anti-synchronization of uncertain time-delayed chaotic neural networks.

**Theorem 1** *If there exist $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $W = W^T > 0$, and $M$ such that*

$$\begin{bmatrix} [1,1] & P\bar{A} & W & PB & P\bar{B} & I & 0 \\ \bar{A}^T P & -R & -W & 0 & 0 & 0 & I \\ W & -W & -\frac{1}{\tau}Q & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & -I & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -\frac{1}{L_f^2} I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & -\frac{1}{L_g^2} I \end{bmatrix} < 0, \tag{5}$$

*where*

$$[1,1] = A^T P + PA + M + M^T + R + \tau Q,$$

*then the adaptive anti-synchronization for time-delayed chaotic neural networks is achieved under the control input*

$$u(t) = P^{-1} M(\hat{x}(t) + x(t)) - \sum_{k=1}^{p} \Phi_k(x(t))\hat{\theta}_k(t) - \sum_{l=1}^{q} \Psi_l(x(t-\tau))\hat{\phi}_l(t) \tag{6}$$

*and the adaptive laws*

$$\dot{\hat{\theta}}_k(t) = \Gamma \Phi_k^T(x(t)) P(\hat{x}(t) + x(t)) \quad (k = 1, \ldots, p), \tag{7}$$

$$\dot{\hat{\phi}}_l(t) = \Upsilon \Psi_l^T(x(t-\tau)) P(\hat{x}(t) + x(t)) \quad (l = 1, \ldots, q), \tag{8}$$

*where $\Gamma$ and $\Upsilon$ are positive definite matrices for design.*

*Proof* The closed-loop anti-synchronization error system with the control input $u(t) = K(\hat{x}(t) + x(t)) - \sum_{k=1}^{p} \Phi_k(x(t))\hat{\theta}_k(t) - \sum_{l=1}^{q} \Psi_l(x(t-\tau))\hat{\phi}_l(t)$, where $K \in R^{n\times n}$ is the gain matrix of the controller, can be written as

$$\begin{aligned} \dot{e}(t) &= (A + K)e(t) + \bar{A}e(t-\tau) + B(f(\hat{x}(t)) + f(x(t))) + \bar{B}(g(\hat{x}(t-\tau)) + g(x(t-\tau))) \\ &\quad - \sum_{k=1}^{p} \Phi_k(x(t))\tilde{\theta}_k(t) - \sum_{l=1}^{q} \Psi_l(x(t-\tau))\tilde{\phi}_l(t), \end{aligned} \tag{9}$$

where $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta_k$ and $\tilde{\phi}_l(t) = \hat{\phi}_l(t) - \phi_l$. Consider the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) &= e^T(t)Pe(t) + \int_{-\tau}^{0}\int_{t+\beta}^{t} e^T(\alpha)Qe(\alpha)d\alpha d\beta + \int_{-\tau}^{0} e^T(t+\sigma)Re(t+\sigma)d\sigma \\ &\quad + \left[\int_{-\tau}^{0} e(t+\sigma)d\sigma\right]^T W \left[\int_{-\tau}^{0} e(t+\sigma)d\sigma\right] + \sum_{k=1}^{p} \tilde{\theta}_k^T(t)\Gamma^{-1}\tilde{\theta}_k(t) \\ &\quad + \sum_{l=1}^{q} \tilde{\phi}_l^T(t)\Upsilon^{-1}\tilde{\phi}_l(t). \end{aligned} \tag{10}$$

Its time derivative along the trajectory of (9) is

$$\begin{aligned}
\dot{V}(t) &= \dot{e}(t)^T Pe(t) + e^T(t)P\dot{e}(t) + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)d\sigma \\
&\quad + e(t)^T Re(t) - e^T(t-\tau)Re(t-\tau) + [e(t)-e(t-\tau)]^T W\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] \\
&\quad + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T W[e(t)-e(t-\tau)] + 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Gamma^{-1}\dot{\hat{\theta}}_k(t) \\
&\quad + 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Upsilon^{-1}\dot{\hat{\phi}}_l(t) \\
&= e^T(t)[A^T P + PA + PK + K^T P]e(t) + e^T(t)P\bar{A}e(t-\tau) + e^T(t-\tau)\bar{A}^T Pe(t) \\
&\quad + e^T(t)PB(f(\hat{x}(t)) + f(x(t))) + (f(\hat{x}(t)) + f(x(t)))^T B^T Pe(t) \\
&\quad + e^T(t)P\bar{B}(g(\hat{x}(t-\tau)) + g(x(t-\tau))) + (g(\hat{x}(t-\tau)) + g(x(t-\tau)))^T \bar{B}^T Pe(t) \\
&\quad - 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Phi_k^T(x(t))Pe(t) - 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Psi_l^T(x(t-\tau))Pe(t) + \tau e^T(t)Qe(t) \\
&\quad - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)d\sigma + e(t)^T Re(t) - e^T(t-\tau)Re(t-\tau) + [e(t)-e(t-\tau)]^T W \\
&\quad \times \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T W[e(t)-e(t-\tau)] + 2\sum_{k=1}^{p}\tilde{\theta}_k^T(t)\Gamma^{-1}\dot{\hat{\theta}}_k(t) \\
&\quad + 2\sum_{l=1}^{q}\tilde{\phi}_l^T(t)\Upsilon^{-1}\dot{\hat{\phi}}_l(t). \qquad (11)
\end{aligned}$$

Since the activation function vectors $f(x(t))$ and $g(x(t))$ of the Hopfield neural networks and the cellular neural networks are odd functions, for each $x, y \in R^n$, it is easy to have

$$\begin{aligned}
\|f(x(t)) + f(y(t))\| &\le L_f\|x(t) + y(t)\|, \\
\|g(x(t)) + g(y(t))\| &\le L_g\|x(t) + y(t)\|, \qquad (12)
\end{aligned}$$

from the Lipschitz conditions of $f(x(t))$ and $g(x(t))$. If we use the inequality $X^T Y + Y^T X \le X^T \Lambda X + Y^T \Lambda^{-1} Y$, which is valid for any matrices $X \in R^{n\times m}$, $Y \in R^{n\times m}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in R^{n\times n}$, we have

$$\begin{aligned}
&e^T(t)PB(f(\hat{x}(t)) + f(x(t))) + (f(\hat{x}(t)) + f(x(t)))^T B^T Pe(t) \\
&\quad \le (f(\hat{x}(t)) + f(x(t)))^T(f(\hat{x}(t)) + f(x(t))) + e^T(t)PBB^T Pe(t) \\
&\quad \le L_f^2(\hat{x}(t) + x(t))^T(\hat{x}(t) + x(t)) + e^T(t)PBB^T Pe(t) \\
&\quad = e^T(t)(L_f^2 I + PBB^T P)e(t) \qquad (13)
\end{aligned}$$

and

$$\begin{aligned} &e^T(t)P\bar{B}(g(\hat{x}(t-\tau))+g(x(t-\tau)))+(g(\hat{x}(t-\tau))+g(x(t-\tau)))^T\bar{B}^TPe(t) \\ &\quad\le (g(\hat{x}(t-\tau))+g(x(t-\tau)))^T(g(\hat{x}(t-\tau))+g(x(t-\tau)))+e^T(t)P\bar{B}\bar{B}^TPe(t) \\ &\quad\le L_g^2(\hat{x}(t-\tau)+x(t-\tau))^T(\hat{x}(t-\tau)+x(t-\tau))+e^T(t)P\bar{B}\bar{B}^TPe(t) \\ &\quad= L_g^2e^T(t-\tau)e(t-\tau)+e^T(t)P\bar{B}\bar{B}^TPe(t). \end{aligned} \tag{14}$$

Using (13) and (14), we obtain

$$\begin{aligned} \dot{V}(t) \le{}& e^T(t)\big[A^TP+PA+PK+K^TP+L_f^2I+PBB^TP+P\bar{B}\bar{B}^TP+\tau Q+R\big]e(t) \\ &+e^T(t)P\bar{A}e(t-\tau)+e^T(t-\tau)\bar{A}^TPe(t)+e^T(t-\tau)\big[L_g^2I-R\big]e(t-\tau) \\ &-\int_{t-\tau}^t e^T(\sigma)Qe(\sigma)d\sigma+[e(t)-e(t-\tau)]^TW\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]+\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]^T \\ &\times W[e(t)-e(t-\tau)]+2\sum_{k=1}^p\tilde{\theta}_k^T(t)\Gamma^{-1}\left[\dot{\hat{\theta}}_k(t)-\Gamma\Phi_k^T(x(t))Pe(t)\right] \\ &+2\sum_{l=1}^q\tilde{\phi}_l^T(t)\Upsilon^{-1}\left[\dot{\hat{\phi}}_l(t)-\Upsilon\Psi_l^T(x(t-\tau))Pe(t)\right]. \end{aligned}$$

Using the inequality [28]

$$\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]^TQ\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]\le\tau\int_{t-\tau}^t e(\sigma)^TQe(\sigma)d\sigma, \tag{15}$$

we have

$$\begin{aligned} \dot{V}(t) \le{}& e^T(t)\big[A^TP+PA+PK+K^TP+L_f^2I+PBB^TP+P\bar{B}\bar{B}^TP+\tau Q+R\big]e(t) \\ &+e^T(t)P\bar{A}e(t-\tau)+e^T(t-\tau)\bar{A}^TPe(t)+e^T(t-\tau)\big[L_g^2I-R\big]e(t-\tau) \\ &-\frac{1}{\tau}\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]^TQ\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]+[e(t)-e(t-\tau)]^TW\left[\int_{t-\tau}^t e(\sigma)d\sigma\right] \\ &+\left[\int_{t-\tau}^t e(\sigma)d\sigma\right]^TW[e(t)-e(t-\tau)]+2\sum_{k=1}^p\tilde{\theta}_k^T(t)\Gamma^{-1}\big[\dot{\hat{\theta}}_k(t)-\Gamma\Phi_k^T(x(t))Pe(t)\big] \\ &+2\sum_{l=1}^q\tilde{\phi}_l^T(t)\Upsilon^{-1}\big[\dot{\hat{\phi}}_l(t)-\Upsilon\Psi_l^T(x(t-\tau))Pe(t)\big] \\ ={}&\begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^t e(\sigma)d\sigma \end{bmatrix}^T\begin{bmatrix} (1,1) & P\bar{A} & W \\ \bar{A}^TP & (2,2) & -W \\ W & -W & -\frac{1}{\tau}Q \end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^t e(\sigma)d\sigma \end{bmatrix}+2\sum_{k=1}^p\tilde{\theta}_k^T(t)\Gamma^{-1} \\ &\times\big[\dot{\hat{\theta}}_k(t)-\Gamma\Phi_k^T(x(t))Pe(t)\big]+2\sum_{l=1}^q\tilde{\phi}_l^T(t)\Upsilon^{-1}\big[\dot{\hat{\phi}}_l(t)-\Upsilon\Psi_l^T(x(t-\tau))Pe(t)\big], \end{aligned} \tag{16}$$

where

$$(1,1) = A^T P + PA + PK + K^T P + L_f^2 I + PBB^T P + P\bar{B}\bar{B}^T P + \tau Q + R,$$
$$(2,2) = L_g^2 I - R.$$

If the adaptive laws (7)–(8) are used and the following matrix inequality is satisfied

$$\begin{bmatrix} (1,1) & P\bar{A} & W \\ \bar{A}^T P & (2,2) & -W \\ W & -W & -\frac{1}{\tau}Q \end{bmatrix} < 0, \tag{17}$$

we have

$$\dot{V}(t) < 0. \tag{18}$$

This guarantees

$$\lim_{t\to\infty} e(t) = 0 \tag{19}$$

from Lyapunov-Krasovskii stability theory. From Schur complement, the matrix inequality (17) is equivalent to

$$\begin{bmatrix} \{1,1\} & P\bar{A} & W & PB & P\bar{B} & I & 0 \\ \bar{A}^T P & -R & -W & 0 & 0 & 0 & I \\ W & -W & -\frac{1}{\tau}Q & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & -I & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -\frac{1}{L_f^2}I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & -\frac{1}{L_g^2}I \end{bmatrix} < 0, \tag{20}$$

where

$$\{1,1\} = A^T P + PA + PK + K^T P + R + \tau Q.$$

If we let $M = PK$, (20) is equivalently changed into the LMI (5). Then the gain matrix of the control input $u(t)$ is given by $K = P^{-1}M$. This completes the proof. □

*Remark 1* Various efficient convex optimization algorithms can be used to check whether the LMI (5) is feasible. In this paper, in order to solve the LMI, we utilize MATLAB LMI Control Toolbox [29], which implements state-of- the-art interior-point algorithms.

## 4 Numerical Example

Consider the following time-delayed chaotic Hopfield neural network [11]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & -\kappa_1 \\ -5 & 1.5 \end{bmatrix} \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix}$$
$$+ \begin{bmatrix} -1.5 & -0.1 \\ -\kappa_2 & -1 \end{bmatrix} \begin{bmatrix} \tanh(x_1(t-1)) \\ \tanh(x_2(t-1)) \end{bmatrix}, \tag{21}$$

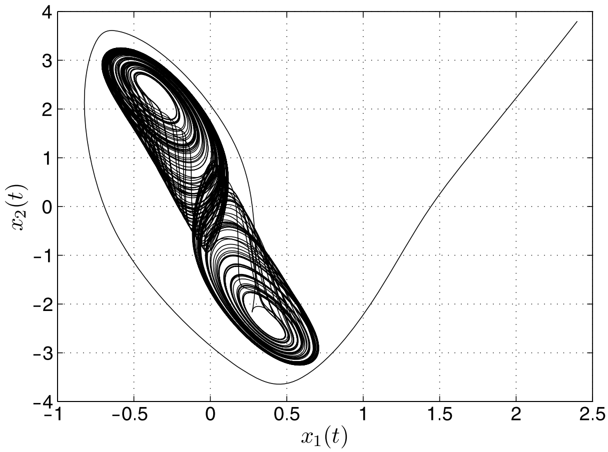



**Fig. 1** The chaotic behavior of the drive system

where $x_i(t)$ $(i=1,2)$ is the state variable of the neural network (21). Parameters $\kappa_1$ and $\kappa_2$ are assumed unknown. The neural network (21) is rewritten as

$$\begin{aligned}\dot{x}(t) = Ax(t) + \bar{A}x(t-1) + Bf(x(t)) + \bar{B}g(x(t-1)) + \Phi_1(x(t))\theta_1 \\ + \Psi_1(x(t-1))\phi_1,\end{aligned} \tag{22}$$

where

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 & 0 \\ -5 & 1.5 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} -1.5 & -0.1 \\ 0 & -1 \end{bmatrix},$$

$$f(x(t)) = \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix}, \qquad g(x(t-1)) = \begin{bmatrix} \tanh(x_1(t-1)) \\ \tanh(x_2(t-1)) \end{bmatrix},$$

$$\theta_1 = \kappa_1, \qquad \phi_1 = \kappa_2,$$

$$\Phi_1(x(t)) = \begin{bmatrix} -\tanh(x_2(t)) \\ 0 \end{bmatrix}, \qquad \Psi_1(x(t-1)) = \begin{bmatrix} 0 \\ -\tanh(x_1(t-1)) \end{bmatrix}.$$

For the numerical simulation, we use the following parameters:

$$\kappa_1 = 0.1, \qquad \kappa_2 = 0.2, \qquad \Gamma = 50, \qquad \Upsilon = 200, \qquad L_f = L_g = 1.$$

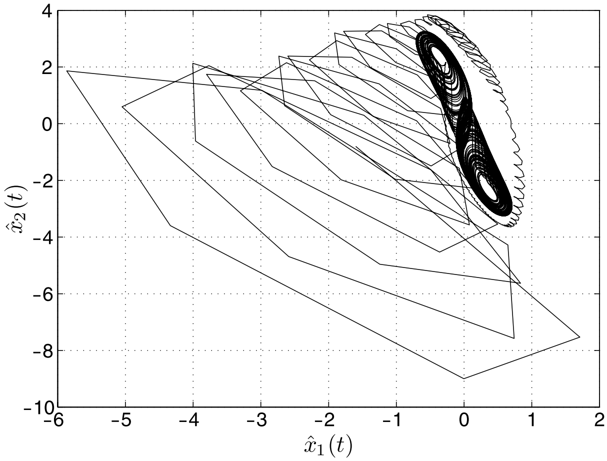
Applying Theorem 1 to the neural network (22) yields

$$P = \begin{bmatrix} 0.3896 & 0.1282 \\ 0.1282 & 0.1047 \end{bmatrix}, \qquad M = \begin{bmatrix} -2.9711 & 8.2550 \\ -8.0979 & -3.1455 \end{bmatrix}.$$

Phase-plane trajectories for drive and response systems are shown in Figs. 1 and 2, respectively, when the initial conditions are given by

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2.4 \\ 3.8 \end{bmatrix}, \qquad \begin{bmatrix} \hat{x}_1(0) \\ \hat{x}_2(0) \end{bmatrix} = \begin{bmatrix} -1.6 \\ -0.8 \end{bmatrix}, \qquad \hat{\theta}_1(0) = 1, \qquad \hat{\phi}_1(0) = 0.5. \tag{23}$$

**Fig. 2** The chaotic behavior of the response system



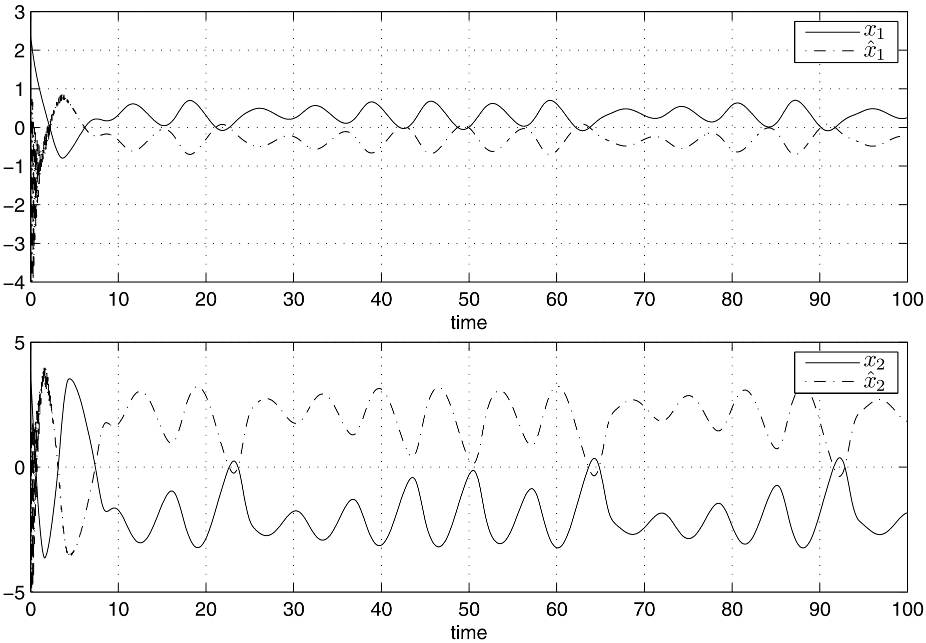




**Fig. 3** State trajectories

Figure 3 shows state trajectories for drive and response systems. From Fig. 3, it can be seen that drive and response systems are indeed achieving chaos anti-synchronization. The anti-synchronization error between drive and response systems is given in Fig. 4. It shows that the anti-synchronization error converges to zero asymptotically. The estimates $\hat{\theta}_1(t)$ and $\hat{\phi}_1(t)$ of the unknown parameters $\theta_1$ and $\phi_1$ are illustrated at Figs. 5 and 6, respectively. These figures show that the estimates $\hat{\theta}_1(t)$ and $\hat{\phi}_1(t)$ approach rapidly to target values 0.1 and 0.2, respectively.

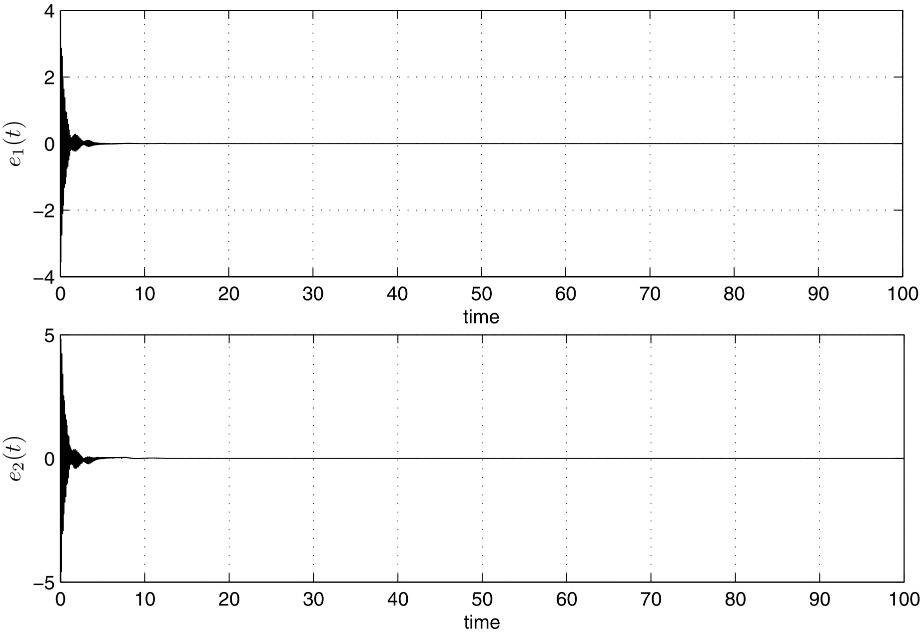



**Fig. 4** Anti-synchronization errors

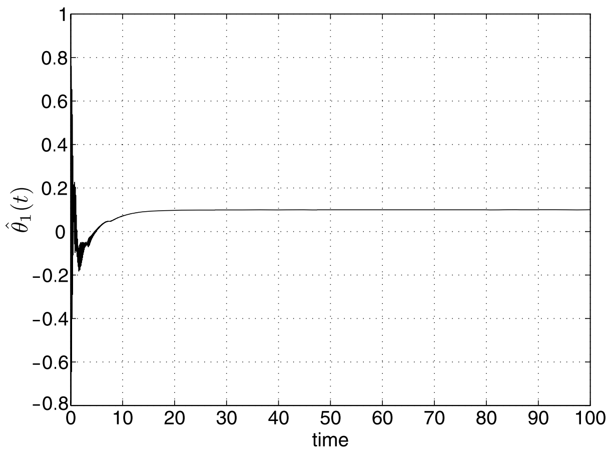
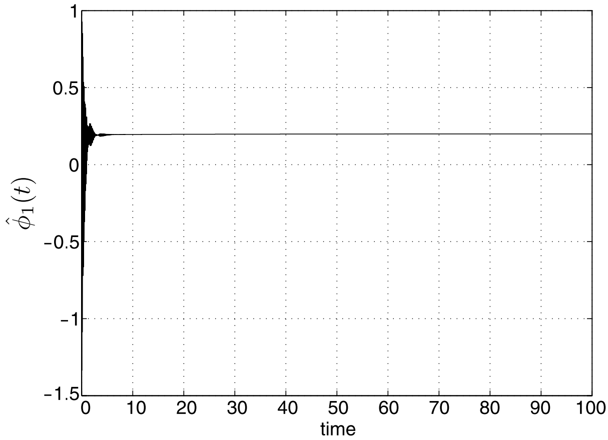



**Fig. 5** The estimate value $\hat{\theta}_1(t)$ of parameter $\theta_1$

## 5 Conclusion

In this paper, a new adaptive anti-synchronization scheme for time-delayed chaotic neural networks with unknown parameters is proposed. Based on Lyapunov-Krasovskii stability theory and LMI approach, the proposed controller is designed and an analytic expression of the controller with its adaptive laws of unknown parameters is shown. A simulation example is given to show the effectiveness of the proposed method. It is expected that the proposed method can be extended to studying adaptive anti-synchronization problems for chaotic neural networks with time-varying and distributed delays.

**Fig. 6** The estimate value $\hat{\phi}_1(t)$ of parameter $\phi_1$




## References


1. Pecora, L.M., Carroll, T.L.: Synchronization in chaotic systems. Phys. Rev. Lett. **64**, 821–824 (1996)
2. Chen, G., Dong, X.: From Chaos to Order. World Scientific, Singapore (1998)
3. Zhang, Y., Sun, J.: Chaotic synchronization and anti-synchronization based on suitable separation. Phys. Lett. A **330**, 442–447 (2004)
4. Kim, C.M., Rim, S., Kye, W.H., Ryu, J.W., Park, Y.J.: Anti-synchronization of chaotic oscillators. Phys. Lett. A **320**, 39–49 (2003)
5. Li, C., Liao, X.: Anti-synchronization of a class of coupled chaotic systems via linear feedback control. Int. J. Bifurc. Chaos **16**, 1041–1047 (2006)
6. Idowu, B.A., Vincent, U.E., Njah, A.N.: Anti-synchronization of chaos in nonlinear gyros via active control. J. Math. Control Sci. Appl. **1**, 191–200 (2007)
7. Vincent, U.E., Laoye, J.A.: Synchronization, anti-synchronization and current transport in non-identical chaotic ratchets. Physica A **384**, 230–240 (2007)
8. Al-sawalha, M.M., Noorania, M.S.M.: On anti-synchronization of chaotic systems via nonlinear control. Chaos Solitons Fractals **42**, 170–179 (2009)
9. Mackey, M.C., Glass, L.: Oscillation and chaos in physiological control systems. Science **197**, 287–289 (1977)
10. Farmer, J.D.: Chaotic attractors of an infinite-dimensional dynamical system. Physica D **4**, 366–393 (1982)
11. Lu, H.: Chaotic attractors in delayed neural networks. Phys. Lett. A **298**, 109–116 (2002)
12. Park, J.H., Kwon, O.M.: Guaranteed cost control of time-delay chaotic systems. Chaos Solitons Fractals **27**, 1011–1018 (2006)
13. Chen, B., Liu, X., Tong, S.: Guaranteed cost control of time-delay chaotic systems via memoryless state feedback. Chaos Solitons Fractals **34**, 1683–1688 (2007)
14. Guan, X., Feng, G., Chen, C., Chen, G.: A full delayed feedback controller design method for time-delay chaotic systems. Physica D **227**, 36–42 (2007)
15. Chen, M., Chen, W.H.: Robust adaptive neural network synchronization controller design for a class of time delay uncertain chaotic systems. Chaos Solitons Fractals **41**, 2716–2724 (2009)
16. Zhu, W., Xu, D., Huang, Y.: Global impulsive exponential synchronization of time-delayed chaotic systems. Chaos Solitons Fractals **35**, 904–912 (2008)
17. Liu, X.: Impulsive synchronization of chaotic systems subject to time delay. Nonlinear Anal. Theory Methods Appl. (2009). doi:10.1016/j.na.2009.01.162
18. Wang, Y., Guan, Z.H., Wang, H.O.: Feedback an adaptive control for the synchronization of Chen system via a single variable. Phys. Lett. A **312**, 34–40 (2003)
19. Park, J.H.: Adaptive synchronization of a unified chaotic systems with an uncertain parameter. Int. J. Nonlinear Sci. Numer. Simul. **6**, 201–206 (2005)
20. Yassen, M.T.: Adaptive synchronization of two different uncertain chaotic systems. Phys. Lett. A **337**, 335–341 (2005)
21. Li, S., Xu, W., Li, R.: Synchronization of two different chaotic systems with unknown parameters. Phys. Lett. A **361**, 98–102 (2007)

22. Guo, R.: A simple adaptive controller for chaos and hyperchaos synchronization. Phys. Lett. A **372**, 5593–5597 (2008)
23. Wang, Z.: Anti-synchronization in two non-identical hyperchaotic systems with known or unknown parameters. Commun. Nonlinear Sci. Numer. Simul. **14**, 2366–2372 (2009)
24. Al-sawalha, M.M., Noorani, M.S.M.: Anti-synchronization of chaotic systems with uncertain parameters via adaptive control. Phys. Lett. A **373**, 2852–2857 (2009)
25. Krasovskii, N.N.: Application of Lyapunov's second method for equations with time delay. Prikl. Mat. Mek. **20**, 315–327 (1956)
26. Kolmanovskii, V., Nosov, A.: Stability of Functional Differential Equations. Academic Press, San Diego (1986)
27. Boyd, S., Ghaoui, L.E., Feron, E., Balakrishinan, V.: Linear Matrix Inequalities in Systems and Control Theory. SIAM, Philadelphia (1994)
28. Noldus, E.: Stabilization of a class of distributional convolutional equations. Int. J. Control **41**, 947–960 (1985)
29. Gahinet, P., Nemirovski, A., Laub, A.J., Chilali, M.: LMI Control Toolbox. The Mathworks Inc., Natick (1995)